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No. 981

NEW METHOD OF CALCULATING THE POWER AT ALTITUDE OF

AIRCRAFT ENGINES EQUIPPED WITH SUPERCHARGERS ON

THE BASIS OF TESTS MADE UNDER SEA-LEVEL CONDITIONS

By Marcello Sarracino

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INTRODUCTION

The present article deals with what is considered to
be a simpler and more accurate method of determining, from
the results of bench tests under approved rating condi-
tions, the power at altitude of a supercharged aircraft
engine, without application of correction formulas.

THE CONVENTIONAL METHOD
(Reference 1)

Before proceeding with the description of the proposed
method, we shall briefly summarize the conventional proce-
dure of determining the characteristics at altitude.

The air intake of the engine is connected, by means
of pipes, to a tank of a certain capacity, the "box," com-
municating with the outside through an opening fitted with
a regulating shutter.

The test is made with full engine throttle, the valve
of the "box" being regulated to vary the air-intake pres-
sure and the boost pressure. At the different speeds, the
horsepower of the engine, the boost pressure, and the pres-
sure in the box are observed — the latter being the one
which defines the fictitious altitude of operation.

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Substantially, such a test - besides affording a correlation between horsepower and boost pressure coincident with the "calibration curve" of the engine at different speeds - merely supplies the compression ratio of the supercharger under different operating conditions.

The corrections to be applied to the data obtained, relate either to the supercharger compression ratio or to the power of the engine. In regard to the supercharger compression ratio, the effect of the air-intake temperature is habitually taken into account by means of an experimental formula, originally proposed by Brooks:

$$r_z = r_p \left[1 + 0.00063 \, r_p{}^2 \, (t_p - t_z) \right] \tag{1}$$

where r is the supercharger compression ratio, and t is the air-intake temperature; the subscripts z and p refer to operating conditions at altitude and sea level, respectively.

This procedure affords for each speed and each altitude in standard air, a corresponding corrected boost pressure and, in particular, determination of the rated altitude at the normal boost pressure.

The horsepower formula generally employed, was also originally suggested by Brooks[1]

$$\Pi_z = \Pi_p \left[1 + 0.00063 \, r_p{}^2 \, (t_p - t_z) \right] \left(1 + \frac{760 - p_z}{3500} \right) \sqrt{\frac{T_p}{T_z}} \tag{2}$$

where Π_z is the effective ~~pressure~~ horsepower, and p_z is the atmospheric pressure (in mm Hg).

This formula enables procedure from the power observed in the test to the engine power developed at altitude, and corresponds with the corrected boost pressure.

The first term in formula (2) allows for the rise in compression ratio of the supercharger, and hence, with

[1]
The Italian standard specifies: $\dfrac{529 + t_p}{529 + t_z}$ instead of $\sqrt{\dfrac{T_p}{T_z}}$.

The results are practically the same.

full engine throttle, for the corresponding rise in boost
pressure of the engine with respect to that disposable in
the test; the second term allows for the rise of density
of the charge consequent to the lower air-intake tempera-
ture; and the third term allows for the depression in the
exhaust which manifests itself either by a better volu-
metric efficiency or by a gain in the effective area of the
cycle due to the lowering of the exhaust line.

A similar formula is used to determine the power de-
veloped in flight, on the basis of the known boost pres-
sure p_a, atmospheric pressure p_z, and temperature t_z.
The power Π_0 is taken from the calibration curves re-
duced to zero altitude in standard air, and corrected by

$$\Pi_z = \Pi_0 \sqrt{\frac{T_p}{T_z}} \left(1 + \frac{p_0 - p_z}{3500}\right) \qquad (3)$$

CRITICISM OF PRESENT CORRECTION METHOD

One critical objection to the correction formula used
at present is, that it is applied to the effective horse-
power, while the atmospheric pressure and temperature di-
rectly affect the weight of air consumed by the engine, to
which - as will be shown - the effective power is not di-
rectly proportional.

This inaccuracy of application of the correction for-
mula - if not conducive to great errors while the correc-
tion factors are small (as, for instance, when referring
the power observed on the test bench at zero altitude in
standard air) - will lead to substantial errors if the
correction factors are high, as occurs with the decrease
of power at high altitude.

The inaccurate method of application of the correction
factors is clearly evident in the first term, relating to
the boost pressure.

Formula (2) assumes that the power varies directly
proportional to the boost pressure while, on the contrary,
as is soon, the power varies in relation to the boost
pressure, according to an approximately rectilinear curve,
which does not pass through the point of origin but inter-
sects the negative semiaxis of the ordinate (reference 2).

As regards the factor relating to the depression in
the exhaust source, objection is raised about its struc-
ture, since theory shows and experience proves that such
a corrective term, applied direct to the effective power,
should be - aside from the exhaust pressure - a function
of the indicated mean pressure of the cycle (reference 3).
Moreover, the effect of depression in the exhaust on the
cylinder charge, is greatly influenced and complicated by
the valve overlap between the intake and exhaust phases -
resulting in partial scavenging of the combustion chamber,
which cannot be allowed for with a general formula appli-
cable to any engine.

PRINCIPLE OF A NEW METHOD OF DETERMINING

THE HORSEPOWER AT ALTITUDE

Cognizant of the deficiencies of the present correc-
tion formulas, various authors have recently advanced more
rational and more accurate methods.

Zeyns and Caroselli, of the DVL (reference 2), pro-
posed a method based on the separation of indicated power
and power loss which, although beyond reproach theoretic-
ally, may encounter difficulties in practice, because it
is predicted on the knowledge of the power loss and its
mode of variation with altitude.

In a subsequent report, Zeyns (reference 4) describes
the results of tests made in the DVL on the air consumption
of engines and its dependence on the altitude, noting par-
ticularly the effect of valve overlap between intake and
exhaust cycle, and its consequent phenomena of scavenging
in the combustion chamber. On this occasion the author
observed, without entering into details, how the knowledge
of air consumption might constitute an indication of the
effective power developed by the engine.

This concept forms a real rational basis for the solu-
tion of the problem presented. In fact, since the sur-
rounding atmospheric conditions do not affect - at least,
for the greater part - the engine power directly, but the
air consumed by the engine, it is possible by searching
first for the effect on the latter and then reverting to
the former by means of an experimentally obtained formula,
to get results which will be simpler and easier to apply,
in general, to different types of engines.

On the other hand, the method is relatively easy to apply, since it merely involves the reading of the air consumption, which is easily accomplished with calibrated nozzles and diaphragms or special volumetric counters.

Developing this principle, we will now explain in detail the mode of application of this method.

First, we establish the correlation between air consumption and horsepower, compute for a specified boost pressure the air consumption, and hence, the power developed under any conditions of atmospheric pressure and temperature different from that of the test; suggesting in this connection, several experimental procedures developed in our laboratories for the purpose of facilitating and making more secure the calculation.

Next, we compare the operation at altitude of the supercharger, indicating new criterions for determining the boost pressure obtainable with full throttle at any altitude.

CORRELATION OF AIR CONSUMPTION AND POWER

IN RELATION TO ALTITUDE

Notation

Π effective horsepower

Π_i indicated horsepower

Π_r power absorbed by mechanical friction, not that of accessory drives (except supercharger)

Π_p pumping power corresponding to the power consumed in the intake and exhaust phases

Π_c power absorbed by the supercharger

P weight of air consumed by the engine in unit time

p_a boost pressure

C piston displacement

ΣC total displacement

n   rpm

H   heat value of fuel

μ air-fuel ratio

η_i indicated efficiency

p   atmospheric pressure

t   atmospheric temperature

We have:

$$\Pi = \Pi_i - \Pi_r - \Pi_c \qquad (4)$$

The indicated horsepower is:

$$\Pi_i = K' \frac{P \, H}{\mu} \eta_i \qquad (5)$$

where K' is a constant, the value of which depends on the employed measuring unit.

Hereinafter, the mixture ratio is assumed to be constant, so that H/μ is a constant. The indicated efficiency η_i introduced in equation (5) is equal to the ratio of energy corresponding to the area of the cycle, and that is equivalent to the heat developed by the combustion of the fuel weight inducted into the cylinder. Defined in that manner, due allowance also is made for the effective processes of energy in the intake and the exhaust phases and the area of the cycle included between the two corresponding representative lines.

Following standard practice, this portion of the cycle may be considered separately by introducing the concept of pumping power. In the more general case, pumping power comprises a negative term corresponding to the energy loss due to fluid resistance in the intake and exhaust pipes (the boost pressure becoming equal to that of the exhaust back pressure, would be represented by the negative area of the cycle comprised between the lines of intake and exhaust), and a term representing the energy due to the eventual difference between boost and exhaust back pressure, and which is positive or negative, according to whether the first is greater or smaller than the second.

With this convention, we have:

$$\Pi_i = K' \frac{P H}{\mu} \eta_i ' + \Pi_p \qquad (6)$$

where $\eta_i '$ is computed as if the cycle developed between an equal boost pressure and exhaust back pressure, and with pumping losses discounted.

Then we can put:

$$\Pi_p = K'' \frac{\Sigma C \, n}{2} \alpha \, (p_a - p_s) - \Pi_p^* \qquad (7)$$

where Π_p^* represents the pumping losses when the boost pressure equals that of the exhaust p_s (the same but opposite prefix for atmospheric pressure p), K'' is a constant dependent on the chosen measuring unit, and α is a suitable coefficient.

Coefficient α eventually differs from unity to allow for the fact that the change in mean pressure of the cycle, due to the difference in boost and exhaust pressure, can be different from that of the two pressures because of eventual modification of the intake and exhaust lines with respect to the course resulting when both pressures are equal. Hence,

$$\Pi = K' \frac{P H}{\mu} \eta_i ' + K'' \frac{\Sigma C \, n}{2} \alpha \, (p_a - p_s) - \Pi_p^* + \Pi_r - \Pi_c \qquad (8)$$

The indicated efficiency $\eta_i '$, in close approximation, can be considered a constant with respect to the changes in atmospheric pressure and temperature, in well-defined limits, within which such changes occur in the problem under consideration.

Power Π_r, absorbed by the mechanical friction, can be considered constant by equal P and n.

Pumping power Π_p consists of a term variable in relation to atmospheric pressure and a constant term.

The power absorbed by the supercharger may be expressed with

$$\Pi_c = P \cdot A \qquad (9)$$

where coefficient A, as will be shown later, follows
from an expression of the type

$$A = a \, n_c^a \, \text{func} \left[\frac{P \sqrt{T_1}}{P_1}, \frac{n_c}{\sqrt{T_1}} \right] \qquad (10)$$

where a is a dimensional constant, n_c the supercharger
rpm, and the third term is a function of the two parame-
ters indicated in parentheses.

Assuming a constant ratio of transmission between en-
gine and supercharger, the power absorbed by the super-
charger is primarily a function of the engine rpm and of
the air consumption; secondarily, and in first approxima-
tion in negligible measure, a function of the above two
parameters which depend on atmospheric pressure and tem-
perature.

The correlation between power and air consumption ex-
pressed in equation (8), is experimentally obtained by the
plotting of a series of curves – each with respect to a
constant rpm, and along which the air consumption is made
to change by action on the carburetor valve. The result is
a family of curves similar to those of the calibration, but
with the difference that the air consumption – rather than
the boost pressure – is plotted on the abscissas.

For the foregoing considerations, the only term modi-
fied in equation (8) is that corresponding to the pumping
power, which changes in ratio of the variation of p_s.

If, however, we plot from the curves representative
of Π in relation to P, those representative of

$$\Pi - K'' \frac{\Sigma C \, n}{2} a \, (p_a - p_s)$$

with respect to the mean variable, the latter will be in-
variable with respect to altitude; i.e., atmospheric pres-
sure and temperature.

Since p_a is known from test observations, it merely
requires that a be known in order that this operation
can be carried out.

In point of fact, there are not enough experimental data available to permit a safe assumption of the factor α. Gnam and Kurz of the DVL (reference 5), on the basis of single-cylinder tests, found α to vary, according to the depression in the exhaust, and to so much lesser degree as the mean piston speed was greater.

We, in turn, by evaluation of tests made in the Fiat altitude test chamber on an A-80-RC-41 engine (reference 3), using various readings of the air consumption taken at that time, reconstructed for such engines the horsepower-air consumption curves for a fixed speed (2100 rpm) and different exhaust pressures.

On comparison of two of such curves relative to 500-meter and 4000-meter barometric height, respectively, we obtained α values ranging from 0.92 to 0.67 from the lowest to the highest boost pressure (of 550 to 850 mm Hg)

It is desirable that additional tests be made on different types of engines in the altitude test chamber, in order to obtain other data on the factor in question.

For the present - lacking such data - we assume hereinafter a constant value of α, corresponding with the average of the Fiat test data: α = 0.8.

Figure 1 shows the power plotted against the air consumption of a 14-cylinder, 2-row radial engine at 2200 rpm. For simplicity, it is limited to one speed only; in practice, of course, such curves should be plotted for all speeds of interest. For the purpose of extending the range of variation of air consumption, the test was made with boost-pressure values considerably higher than normal (860 mm Hg), which was possible by utilizing 100-octane fuel instead of the 87-octane fuel specified for this engine.

From the thus-obtained curve which, for the explored range can be considered a straight line, we deduced that shown as a dashed line (fig. 1) by subtraction of the power corresponding to the difference in boost and exhaust back pressure.

The characteristics of the engine on which the tests were made and to which are referred all the other experimental data presented hereinafter, are as follows:

Air cooling

14 cylinders, 2 rows

Total displacement: 38,600 liters

Stroke: 165 mm

Bore: 146 mm

Compression: 6

Intake valve opens: $4°$ A.T.C.

Intake valve closes: $36°$ A.B.C.

Exhaust opens: $31°$ $30'$ B.B.C.

Exhaust closes: $3°$ $30'$ A.T.C.

AIR CONSUMPTION AND POWER AT ALTITUDE

The solution can be divided into three steps:

a) Ascertain the weight of air aspirated in the engine under changing atmospheric pressure and temperature conditions.

b) After the air consumption is computed, it is plotted on a diagram which gives, for a specified speed and in relation to the air consumption

$$II - K'' \frac{\Sigma Q \; n}{2} \; \alpha \; (p_a - p_s)$$

c) Add to this value the power fraction

$$K'' \frac{\Sigma Q \; n}{2} \; \alpha \; (p_a - p_s)$$

relative to the boost and exhaust back pressure incurred under the pertinent operating conditions.

In order to study the effect on the air consumption of

the variables involved, we express P as function of these variables. We can put

$$P = \frac{\Sigma C\ n}{2} \lambda_r\ \gamma_a \qquad (11)$$

where

$$\gamma_a = \frac{p_a}{R\ T_a}$$

is the specific weight of air at pressure p_a and temperature T_a in the delivery pipes directly before the intake valves, and

λ_r is the volumetric efficiency

For the calculation of γ_a, the boost pressure p_a is known, but not temperature T_a, which differs from that of the supercharger intake due to the heat changes in the carburetor, in the supercharger, and in the inlet pipes.

The volumetric efficiency λ_r is defined as the ratio of air weight effectively inducted into the cylinder to that of the cylinder volume at pressure p_a and temperature T_a.

The cylinder charge is essentially affected by three factors: the pressure drop of air due to resistance encountered in entering the cylinder, the temperature rise of the fresh charge due to the exchange of heat with the cylinder wall, and the loss or gain in volume available, due to the compression and expansion of the residuary gases upon admission of the fresh charge. Hence,

$$\lambda_r = \lambda_{r(p)}\ \lambda_{r(T)}\ \lambda_{r(s)} \qquad (12)$$

For our purposes it is interesting to know how the atmospheric pressure and temperature affect $\left(\lambda_r \not\subset \gamma_a\right)$ or, in other words, since p_a is known,

$$\lambda_r\ \frac{1}{T_a}$$

The atmospheric pressure acts essentially on $\lambda_{r(s)}$

which, in first approximation at least, can be considered independent of the temperature.

The air-intake temperature acts essentially on T_a, and $\lambda_{r(T)}$ which, however, may be considered independent of the pressure; then on $\lambda_{r(p)}$ which, approximately, can also be considered independent of the atmospheric pressure.

Postponing the detailed study of $\lambda_{r(s)}$ and its dependence on atmospheric pressure, for the time being, we note for the remaining terms of the product

$$\lambda_r \frac{1}{T_a}$$

that, so far as we are concerned, the knowledge of their absolute values is unnecessary and that only relative values are needed — or, what amounts to the same thing — the law of variation of air consumption in relation to the air-intake temperature under otherwise identical operating conditions.

In accord with commonly accepted assumptions, it may be supposed that the air consumptions are in the inverse ratio of the square roots of the absolute intake temperatures. This assumption is established when applying the original experimental law commonly accepted for the power to the air consumption.

Recent tests at the DVL, referred to by Zeyns in his report, indicated a relationship between air consumption and temperature in the inverse ratio of the square root. This point would seem, then, to be even clearer, once ample data from altitude-chamber tests are available. Some of the more general formulas can be obtained by referring to the temperature behind the supercharger as well as the air-intake temperature, thus eliminating the variable of the temperature rise in the supercharger, which may be dissimilar for every engine, according to the characteristics of operation and supercharger efficiency itself.

To evaluate $\lambda_{r(s)}$, the phenomena observed during the cylinder charge must be taken into account, due to the fact that the pressure of the fresh charge differs from that of the burned residuary gas. Under the assumption that there is no valve overlap between the closing of the ex-

haust and the opening of the air intake, the cylinder volume occupied or left free by the residuary gases due to expansion or compression, can be computed with

$$\Delta V = \frac{C}{\epsilon - 1} \frac{1}{m} \left(1 - \frac{p_s}{p_a}\right) \qquad (13)$$

hence

$$\lambda_{r(s)} = 1 + \frac{1}{(\epsilon - 1) \ m} \left(1 - \frac{p_s}{p_a}\right) \qquad (14)$$

where

    ΔV volume of combustion chamber left free or filled with residuary gases

    ϵ compression ratio

    m exponent of polytropic process of compression or expansion of exhaust gases

    p_s pressure in exhaust (coincident with ambient pressure p)

The other symbols are known.

In reality, the process does not correspond with the simplified scheme which affords the above formula, and is usually complicated by other than negligible valve overlap between the intake and the exhaust phases commonly existing in modern engines which induce real scavenging phenomena in the combustion chamber.

In any case, it is plausible and justified to admit that $\lambda_{r(s)}$ is a general function of the ratio p_s/p_a of exhaust to boost pressure. In fact, the DVL tests mentioned by Zeyns and Caroselli were conducted with this form of the function. These experiments showed that $\lambda_{r(s)}$ is largely dependent upon the valve overlap and, specifically, that it is not a linear function of p_s/p_a, except for small overlap values.

For the final proposals in the present study, it is of great importance to be able to determine for the test engine the values of $\lambda_{r(s)}$ at different ratios p_s/p_a, even when not concerned with an altitude test plant.

Tho path of tho $\lambda_{r(s)}$ function corresponding to tho values $\dfrac{p_s}{p_a} > 1$, is obtained readily from simple bench tests by throttling the intake and by observing the air consumption; it affords a short path for values of $\dfrac{p_s}{p_a} < 1$, extending almost to the maximum permissible boost pressure. The corresponding $\lambda_{r(s)}$ values for smaller ratios of p_s/p_a encountered at high altitude, can be obtained if altitude equipment is available, by extrapolation of the path of the computed curve.

Zeyns, using the results he obtained in the altitude test chamber, presents a series of curves for different values of valve overlap which may serve as a guide in such extrapolation. With proper experimental procedure, this extrapolation can be equally facilitated. It is clear that if the boost pressure could be raised beyond the normal value to full throttle at sea level, the problem would yield the same result, since the minimum value of p_s/p_a corresponds inversely to the compression ratio of the supercharger which — tho slight difference due to air-intake temperature being disregarded — is a constant with altitude.

This method of operation is not possible, generally, because, aside from overstressing the engine parts, the phenomena of detonation become so severe as to make engine operation precarious and, in any case, so disturbing as, probably, to influence tho appraisal of tho very phenomenon that we want to bring into evidence. However, oven so, tho path necessary for tho extrapolation can bo reduced to a minimum through tho use of a higher octane fuel than specified for tho engine and by pushing tho boost pressure, if not to full throttle, at least to higher values than normally used.

Tho rost of tho extrapolation can bo greatly facilitated by tho following expedient: secure $\lambda_{r(s)}$ by driving tho engine from an electric rotor, tho ignition being cut out and tho boost pressure raised to the highest value obtainable. Under those conditions, tho engine-operating conditions are naturally unlike those prevailing normally; so also are tho effects of tho phenomenon in question, since tho nature and temperature of tho residuary gases are different and, in general, the temperature of tho en-

gine parts is different. In spite of this, it is note-
worthy that, for shape at least, the curves that define
the function $\lambda_{r(s)}$ are similar, so that the second can
serve as guide to the first in the necessary extrapolation.
This method merely calls for an electric motor of adequate
horsepower.

Figures 2 and 3 illustrate various experimental re-
sults obtained at Guidonia, on the previously described
engine.

Figure 2 gives the air consumption in relation to
boost pressure, for 2200 rpm. Figure 3 shows the coeffi-
cient $\lambda_{r(s)}$ plotted against the pressure ratio p_s/p_a.
$\lambda_{r(s)}$ is rectilinear over a wide range of p_s/p_a, and
diverges only at very high p_s/p_a.

The straight course of $\lambda_{r(s)}$ is justified for the
engine tested with the smallest valve overlap; in any case,
the slope of the straight line is better than that obtained
from (14) by assuming a plausible value per meter.

The determination of $\lambda_{r(s)}$ was extended to much lower
values of ratio p_s/p_a, which were obtained by raising the
boost pressure almost to that at sea level, using 100-
octane fuel in order to avoid detonation. Figure 3 shows
further, the values for $\lambda_{r(s)}$ as obtained when running
the engine with an electric motor. These points fall, in
very best approximation, on the particular $\lambda_{r(s)}$ curve
obtained in normal operation, thus confirming the correct-
ness of the previously suggested experimental method.

It is to be noted that the coincidence of the values
of $\lambda_{r(s)}$, achieved in both cases, demonstrates that the
phenomenon is not affected by the residuary gas tempera-
ture. However, this result merits further confirmation.

The calculation of the air consumption P for a
specified speed at any p_a, p, and t can now be carried
out.

With $P_{(p)}$ and $T_{(p)}$ as the absolute atmospheric
pressure and temperature at which the tests were made, and
P^* as the air consumption observed in correspondence with
the value of the boost pressure p_a^* equal to the atmos-

pheric pressure of the test $p_{(p)}$, we find

$$P = P* \frac{p_a}{p_a^*} \lambda_{r(s)} \tau \tag{15}$$

The coefficient $\lambda_{r(s)}$ is that relative to the value of ratio p_s/p_a, while τ allows for the effect of temperature on the filling or charge and which, in accord with previous arguments can be expressed by the square root of the inverse ratio of the absolute air-intake temperature

$$\tau = \sqrt{\frac{T_p}{T}} \tag{16}$$

The procedure can be applied forthwith to the calculation of the change in air consumption with altitude in standard air.

Up to the rated altitude the boost pressure remains constant; beyond, however, it decreases in accord with a law defined by the characteristics of the supercharger and of the air intake and can be computed by the methods indicated hereinafter.

However, an approximation can be made and so the calculation for above boost pressure, avoided. With subscript n denoting the quantity relative to rated altitude, we have at above this altitude:

$$P_z = P_n \frac{p_z}{p_n} \frac{r_z}{r_n} \sqrt{\frac{T_n}{T_z}} \frac{(\lambda_{r(s)})_z}{(\lambda_{r(s)})_n} \tag{17}$$

If, in the approximation, Brooks' formula is used to represent the variation of r with the induction temperature, we find that the law expressed by (17) is much closer to that of the density.

For illustration, the expression

$$\frac{r_z}{r_0} \frac{p_z}{p_0} \sqrt{\frac{T_0}{T_z}}^{a)}$$

is developed in relation to the altitude in standard

a) The slight variation of $\lambda_{(r)s}$ at above rated altitude can be disregarded.

air for a compression ratio of $r_o = 1.7$. As shown in table I, the values obtained are much closer to those of the relative density δ.

TABLE I

Altitude in stand- ard air	δ	$\frac{p_z}{p_0} \left[1+0.00063 \; r_o{}^2 \; (t_o-t_z) \right] \sqrt{\frac{T_0}{T_z}}$
0	1	1
1.000	0.9075	0.9077
2.000	.8216	.8217
3.000	.7421	.7422
4.000	.6686	.6680
5.000	.6008	.5992
6.000	.5384	.5363
7.000	.4811	.4781
8.000	.4286	.4246
9.000	.3805	.3758
10.000	.3368	.3312
11.000	.2969	.2909

APPLICATION OF METHOD TO POWER DEVELOPED IN FLIGHT

In the general case, previously discussed, we have the problem of determining the power developed in flight on the basis of data supplied by aircraft instruments: engine speed, boost pressure, atmospheric pressure, and temperature.

By the indicated method, based upon the data observed in test at sea level under specified conditions of atmospheric pressure and temperature, and for different speeds, we compute the air consumption - and from it, the power developed - through the consumption-power curve and application of the correction for the difference in boost and exhaust pressure.

The operations are longer than with the method of passing directly from the calibration curves obtained at sea level to the horsepower at altitude, but they afford a superior approximation, especially in the cruising range, where the actual correction formulas are usually far from accurate.

Aside from that, the knowledge of air consumption under those conditions can be useful. We stress the possibility of calculating, in this manner, the fuel consumption which must be maintained at altitude in order to achieve a prearranged air-fuel ratio.

The mathematical operations can be almost completely eliminated and the approximation made even closer if the quantity of air inducted in the engine is observed direct in flight by means of a nozzle.

This practice merits serious consideration since it does not present special difficulties and the knowledge of air consumption can be very useful, as already stated, through the possibility of affording a check on the mixture ratio by way of the contemporary measurement of fuel consumption.

VARIATION OF COMPRESSION RATIO OF SUPERCHARGER WITH

ALTITUDE AND TEST METHOD OF PREDICTION

The induction temperature varies with the compression ratio of the supercharger.

This fact is generally taken into account by applying to the compression ratio observed in bench tests, a correction factor suggested by Brooks at:

$$r_z = r_p \left[1 + 0.00063 \, r_p^2 \, (t_p - t_z) \right] \qquad (1a)$$

This is an experimental correction formula.

Theoretically, a formula can be obtained which brings out the effect of intake temperature on the compression ratio by equalizing the effective prevalence which is obtained by the two different intake temperatures in question. We obtain:

$$r_z = r_p \left[1 + \left(\frac{T_p}{T_z} - 1 \right) \left(1 - \left(\frac{1}{r_p} \right)^{\frac{m-1}{m}} \right) \right]^{\frac{m}{m-1}} \qquad (18)$$

where m is the exponent of the polytropic process, according to which the compression operates.

A similar formula proposed by Zorns and Caroselli pre-supposes constancy of the adiabatic prevalence.

On comparison of the theoretical formulas with those by Brooks, it is found that the obtained values are in good agreement for compression ratios up to 2.5 and for altitudes up to 6000 meters, whereas above that, the discrepancies become considerable.

Formula (1a) is applicable only on the assumption that the supercharger operates constantly at the same point of its characteristic manometric curve - a hypothesis not generally satisfied by changing altitude, as shown later on.

The use of correction formulas can be avoided by making use of the experimental method suggested here, and which derives directly from the conditions of similitude for centrifugal superchargers developed by R. S. Capon and G. V. Brooke (reference 5).

The cited authors effected a dimensional analysis of parameters on which the characteristics of a centrifugal supercharger depend. Disregarding the effects of distortion of the supercharger casing, the heat flow through the casing and, in the range of normal designs, the scale effect, we have:

$$\frac{p_2}{p_1}, \ \frac{\rho_2}{\rho_1}, \ \frac{T_2}{T_1}, \ \frac{\Pi_c}{\frac{P}{g} \ n_c{}^2 \ D^2}, \ \eta_a$$

$$= \text{func.} \left[\frac{P/g}{D^2 \sqrt{p_1 \rho_1}}, \ n_c \ D \ \sqrt{\frac{\rho_1}{p_1}} \right] \quad (19)$$

where the bracketed terms indicate nondimensional variables or, not considering the constant terms:

$$\frac{p_2}{p_1}, \ \frac{\rho_2}{\rho_1}, \ \frac{T_2}{T_1}, \ \frac{\Pi_c}{P \ n_c{}^2 \ D^2}, \ \eta_a = \text{func} \left[\frac{P \sqrt{T_1}}{p_1}, \ \frac{n_c}{\sqrt{T_1}} \right] \quad (20)$$

where

p, ρ, and T are, respectively, the absolute pressure, density, and temperature at the super-charger intake and delivery

n_c rpm of the supercharger rotor

Π_c power required to operate the rotor

η_a adiabatic efficiency

D   linear dimension of a supercharger related to a family of geometrically similar superchargers (for instance, diameter of rotor)

In the application of these relations of similitude to the operation, at altitude, of a supercharger attached to the engine, the quantity referring to the test run at sea level is denoted by subscript p, and to that at altitude Z, by z.

If it were possible to realize the two equalities

$$\frac{n_z}{T_{1(z)}} = \frac{n_p}{\sqrt{T_{1(p)}}}; \qquad P_z \frac{\sqrt{T_{1(z)}}}{p_{1(z)}} = P_p \frac{\sqrt{T_{1(p)}}}{p_{1(p)}} \qquad (21)$$

simultaneously, the supercharger would operate under conditions of similitude and then the compression ratio as well as the other characteristics observed in the ground test, would be equal, at altitude, to those for n_z.

In reality, the two conditions of similitude are impossible to achieve independent of one another because the supercharger is connected to the engine; hence its conditions of operation are linked with those of the engine.

The first condition of similitude can be expressed in the form:

$$\alpha = \frac{\dfrac{n_z}{\sqrt{T_{1(z)}}}}{\dfrac{n_p}{\sqrt{T_{1(p)}}}} = 1 \qquad (22)$$

We verify the value which the ratio assumes in this case:

$$\beta = \frac{\left(\dfrac{P\sqrt{T_1}}{p_1}\right)_z}{\left(\dfrac{P\sqrt{T_1}}{p_1}\right)_p} \qquad (23)$$

so that if the second condition of similitude were also satisfied, the result would be equal to 1. We can put

$$P = \frac{\Sigma C \; n}{2} \; \lambda_r \; \gamma_a = \frac{\Sigma C \; n}{2} \; \lambda_{r(p)} \; \lambda_{r(s)} \; \frac{p_a}{R \; \dfrac{T_a}{\lambda_{r(T)}}} \qquad (24)$$

The temperature $\dfrac{T_a}{\lambda_{r(T)}}$ of the fresh charge in the cylinder differs from the delivery temperature T_z of the supercharger, as a result of the heat transfer between mixture and supercharger walls, and we can retain a certain function $f(T_z)$ of it:

$$P = \frac{\Sigma C \; n}{2} \; \lambda_{r(p)} \; \lambda_{r(s)} \; \frac{p_a}{R \; f \; (T_z)} \qquad (25)$$

Function $f(T_z)$ presumably, shall be such that the ratio between the two different altitude values can be retained between the ratio of T_z and the ratio of $\sqrt{T_z}$.

The substitution of P in the formula for β affords:

$$\beta = \frac{n_z}{n_p} \; \frac{(\lambda_{r(p)})_z}{(\lambda_{(r)p})_p} \; \frac{(\lambda_{r(s)})_z}{(\lambda_{r(s)})_p} \; \frac{\left(\dfrac{\sqrt{T_1}}{f(T_z)}\right)_z}{\left(\dfrac{\sqrt{T_1}}{f(T_z)}\right)_p} \; \frac{\left(\dfrac{p_z}{p_1}\right)_z}{\left(\dfrac{p_z}{p_1}\right)_p} \qquad (26)$$

where p_z is the pressure at delivery, coincident with that of the boost, and p_1 the induction pressure coincident with the atmospheric pressure in operation at altitude and distinct from it; coinciding instead with the pressure in the "box" under test conditions.

Assuming $\dfrac{(\lambda_{r(p)})_z}{(\lambda_{r(p)})_p} = 1$, which is justified by equality of $p/\sqrt{T_1}$, and bearing in mind that

$$\frac{n_z}{n_p} = \frac{(\sqrt{T_1})_z}{(\sqrt{T_1})_p}$$

gives

$$\beta = \frac{(\lambda_{r(s)})_z}{(\lambda_{r(s)})_p} \cdot \frac{r_z}{r_p} \cdot \frac{\left(\frac{T_1}{f(T_2)}\right)_z}{\left(\frac{T_1}{f(T_2)}\right)_p} \qquad (27)$$

To check the limits within which the variation of β is contained, the two assumptions relating to $f(T_2)$ are studied separately.

Assumption 1:

$$\frac{f(T_2)_z}{f(T_2)_p} = \frac{(T_2)_z}{(T_2)_p}; \qquad \beta = \frac{(\lambda_{r(s)})_z}{(\lambda_{r(s)})_p} \cdot \frac{r_z}{r_p} \cdot \frac{\left(\frac{T_1}{T_2}\right)_z}{\left(\frac{T_1}{T_2}\right)_p} \qquad (28)$$

It affords:

$$\frac{T_1}{T_2} = \frac{1}{r^{m-1}}$$

where m is the exponent of the polytropic process of compression, and hence

$$\beta = \frac{(\lambda_{r(s)})_z}{(\lambda_{r(s)})_p} \cdot \frac{r_z^{2-m}}{r_p^{2-m}} \qquad (29)$$

Having achieved the first condition of similitude, which experience shows to be that of the greater effect on the compression ratio, we can retain $r_z = r_p$ in first approximation, hence

$$\beta = \frac{(\lambda_{r(s)})_z}{(\lambda_{r(s)})_p} \qquad (30)$$

Assumption 2:

$$\frac{f(T_2)_z}{f(T_2)_p} = \frac{\sqrt{(T_2)_z}}{\sqrt{(T_2)_p}}$$

$$\beta = \frac{(\lambda_{r(s)})_z}{(\lambda_{r(s)})_p} \frac{r_z}{r_p} \frac{\sqrt{\left(\frac{T_1}{T_2}\right)_z} \sqrt{(T_1)_z}}{\sqrt{\left(\frac{T_1}{T_2}\right)_p} \sqrt{(T_1)_p}}$$

$$= \frac{(\lambda_{r(s)})_z}{(\lambda_{r(s)})_p} \frac{r_z^{\frac{3-m}{2}}}{r_p^{\frac{3-m}{2}}} \sqrt{\frac{(T_1)_z}{(T_1)_p}} \qquad (31)$$

and again retaining $r_z = r_p$ in first approximation, we find:

$$\beta = \frac{(\lambda_{r(s)})_z}{(\lambda_{r(s)})_p} \sqrt{\frac{(T_1)_z}{(T_1)_p}}$$

In order to gain an idea of the values that β may assume - referring to the engine on which the experimental data were obtained - the β value was computed over the range from rated altitude of 860 mm Hg boost pressure, equal to 3600 m and zero altitude in standard air. Hence, the two assumptions of $f(T_2)$:

Assumption 1:

$$\beta = \frac{1.096}{1.025} = 1.069$$

Assumption 2:

$$\beta = \frac{1.096}{1.025} \sqrt{\frac{264.6}{288}} = 1.025$$

Ratio β thus assumes values relatively at little variance with unity; provided the first condition of similitude of the supercharger, $\alpha = 1$, has been realized, it automatically and approximately affords the second condition, $\beta = 1$ - at least, for well-proportioned superchargers operating in the flat region of the characteristic manometric curve.

In any case, it will always be possible - at least, theoretically - to put $\beta = 1$, by suitable manipulation of the boost pressure in the bench test, to a value higher

than that of the operation from which we seek the rated altitude. For example, in the above case, we have: boost pressure from which we seek the rated altitude, 850 mm/Hg.

Assumption 1: boost pressure to be reached in bench test, in order to obtain $\beta = 1$, 1342 mm/Hg.

Assumption 2: boost pressure to be reached in test on ground, in order to obtain $\beta = 1$, 993 mm/Hg.

In the first case, the resultant boost pressures are much higher and, in general, not compatible with safe operation; in the second case, however, the values would be obtainable by using a higher octane fuel.

Effecting the determination of the rated altitude by the conventional method, the value of β departs consistently farther from unity, thus introducing a new source of error, which is difficult to check with a correction formula, as the effect on the compression ratio would certainly differ from one supercharger to the next, according to the shape of its characteristic curve and the zone in which the supercharger happens to operate.

To calculate β in such a case, the assumptions for $f(T_2)$, the variations of $\lambda_{r(p)}$ being disregarded, give:

Assumption 1:

$$\beta = \frac{(\lambda_{r(s)})_z}{(\lambda_{r(s)})_p} \frac{r_z^{a-m}}{r_p^{a-m}} \sqrt{\frac{(T_1)_p}{(T_1)_z}}$$

Assumption 2:

$$\beta = \frac{(\lambda_{r(s)})_z}{(\lambda_{r(s)})_p} \frac{r_z^{\frac{a-m}{a}}}{r_p^{\frac{a-m}{a}}}$$

It is noted that in the test – as it is now conducted – conditions are still worse from this standpoint; in fact, in attempting to find by trial, the altitude at which the corrected boost pressure is equal to that fixed beforehand, we operate at much lower effective boost pressure than normal and hence, with a still higher ratio $\dfrac{(\lambda_{(r)s})_z}{(\lambda_{(r)s})_p}$ than

that considered in the preceding example.

The effect of placing the operating points of the supercharger on its characteristic curve, was confirmed by the previously described Fiat altitude-chamber tests on the A-80-RC-41 engine, which manifested a drop in compression ratio by a decrease of pressure in the exhaust.

Application of the new method suggested for determining the rated altitude, derives directly from what has previously been said.

Wishing to determine for a specified speed, the altitude at which a certain boost pressure is re-established, readings were taken with the "box" at different altitudes, realizing for each test, the condition $\alpha = 1$ and, possibly, $\beta = 1$. Then we plot a curve showing the compression ratio in relation to the altitude; the intersection of the curve with that given by the ratio of the prescribed boost pressure and that corresponding to the altitude, gives the rated altitude. Figure 4 shows the results of such experiments.

Although not explicitly stated, it is understood that Capon and Brooks' laws of similitude are applicable to superchargers in which the fluid is air, exclusively.

On the other hand, the new method proposed here can be satisfactorily applied, disregarding divers facts of heat exchange with the outside across the supercharger walls, in which the association of fuel with air is achieved after the supercharger. The method is not applicable, except approximately, to engines provided with aspirated carburetors. The vaporization of fuel has, as noted, a beneficial effect on the compression ratio of the supercharger. The effect of fuel vaporization is translated into a lowering of the intake temperature, a change in specific heat and in the exponent of the adiabatic; and a subtraction of heat during compression, if vaporization is achieved in the supercharger. The results vary according to whether the vaporization occurs before the supercharger or in it, and precisely in the latter case the change in the compression ratio is a minor one.

Changing the air pressure and temperature entering the carburetor, changes the rate of evaporation of the fuel in direct proportion to the difference between saturated vapor tension and partial vapor tension; in this

case, of course, it is no longer possible to assume $r_x = r_p$, since not even the first condition of similitude is satisfied.

Reverting to the engine described in the foregoing for comparing r_z and r_p according to Brooks' formula, $(\lambda_{r(s)})_z / (\lambda_{r(s)})_p$ is assumed to be equal to the originally computed value, and $r_p = 1.7$. Under these conditions, we find:

Approximation 1: $\beta = 1.129$

Approximation 2: $\beta = 1.098$

inversely proportional to the total pressure. In consequence, it is to be expected that the ratios of the fractions of fuel evaporating first will vary, inside and behind the supercharger, with its consequent effects on the compression ratio of the supercharger which, presumably, will be detrimental as the altitude increases.

In point of fact, no data are available by which this phenomenon could be taken into account with any degree of certainty; therefore, as its effect is not of great importance, it may be disregarded in the first approximation, pending experimental confirmation.

To illustrate the effect of fuel evaporation on the compression ratio, in figure 5 are shown the manometric characteristics of the engine supercharger in our experiments with and without evaporation of fuel.

The top curve is that of normal engine operation; the lower curve was obtained by running the engine by electric motor with ignition cut off and gasoline flow interrupted.

APPLICATION OF METHOD AND COMPARISON WITH RESULTS

OBTAINED IN ALTITUDE TEST CHAMBER

As an illustrative example of the suggested method, we computed the curve of the horsepower variation with altitude in standard air on the same engine that furnished all the foregoing experimental data. The air consumption and horsepower developed at the constant speed of 2200 rpm

are shown plotted against the altitude in figure 6.

The rated altitude for normal boost pressure of 860 mm/Hg, was also plotted by the foregoing experimental method (see fig. 4), and the air consumption and horsepower up to this altitude were computed on the basis of constant boost pressure. Above this altitude, the approximate law of change of air consumption is assumed to apply proportionally to the ratio of the density at the altitude in question and that at the rated altitude. It is interesting to compare the obtained results with those made in the altitude test chamber under actual conditions of temperature and pressure at altitude.

In particular, we refer to the tests in the Fiat altitude chamber on the A-80-RC-41 engine which, so far as we know, are the most complete tests made on a supercharged engine.

A comparison of results on other engines should furnish a useful guide to further research.

For the engine to which the new method was applied, we obtained, between altitude 0 and rated altitude of 3600 meters, a 15.3-percent increase in power – equivalent to an average increase of 4.25 percent per 1000 meters.

For the A-80-RC-41 engine in the altitude test chamber, the increase in power amounted to 17.4 percent between altitude 0 and rated altitude of 725 mm/Hg – equivalent to 4100 meters, or a mean increase of 4.25 percent per 1000 meters altitude.

Applying to the power at zero altitude the corrections actually employed for the temperature and the depression of the exhaust, we obtain within average limits of altitude, a mean increase of around 3.50 percent per 1000 meters altitude.

Applying the formula according to present Italian aeronautical standards, as far as they have been established, to the brake-horsepower reading in the test with correction box,

$$\left[1 + 0.00063 \ r_p{}^2 \ (t_p - t_z) \right] \frac{529 + t_p}{529 + t_z} \left(1 + \frac{760 - p_z}{3500} \right)$$

we obtain, always within average altitude limits, for the

A-80-RC-41 engine, an increase in power of only 3 percent per 1000 meters altitude.

The argument now being made should not, however, lead us to conclude that the horsepower computed by the present standards is in every case below the real horsepower. Discounting the fact that results can change from one engine to the next, it should be noted that by the conventional method, the rated altitude is generally overestimated; thus it becomes possible to recover by this method, the loss through the other method.

The proposed method presents still other advantages over the present correction formulas at low boost pressures, where errors always become greater. In fact - always with reference to the altitude-chamber tests on the A-80-RC-41 engine - a comparison of the calibration curve at the normal speed of 2100 rpm, at 4100 meters, in standard air, with that computed by Brooks' formula, discloses a percentage error rising from 4.4 percent to 23.25 percent for a change in boost pressure from 750 mm/Hg to 500 mm/Hg.

CONCLUSION

The method of calculating the characteristics at altitude, of supercharged engines, based on the consumption of air, is a more satisfactory procedure from a logical point of view, than the conventional correction formulas and affords a more accurate calculation of the horsepower at altitude, especially at low boost pressures.

One important value of this method is, that it enables the determination, on the test engine direct, of the effect of the depression in the exhaust on the cylinder filling - which permits the exact appraisal of a factor, in respect to which greater variety of behavior from the different engine parts may be expected.

The method can be refined after adequate research in the altitude test chamber, in order to establish a more accurate average law representing the effect of air-intake temperature on air consumption and the value to be attributed to the variation in pumping power due to the difference between boost and exhaust back pressure.

The correlation between air consumption and horsepower

can be usefully applied to the determination of the power
expected in flight, by observation, with suitable equip-
ment, of the air consumption of the engine. Experiments
for this purpose are under way and will be discussed in
due time.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

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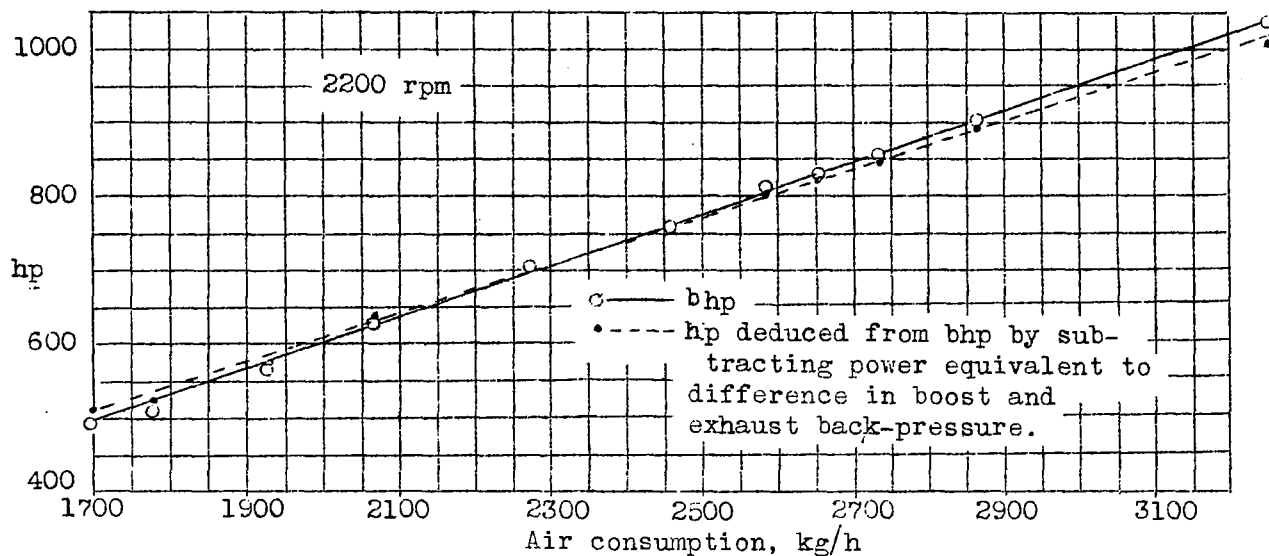   with Inlet Conditions. R. & M. No. 1336, British
      A.R.C., 1930.

2200 rpm

1000

hp

800

600

400

bhp

hp deduced from bhp by sub-
    tracting power equivalent to
difference in boost and
exhaust back-pressure.

1700   1900   2100   2300   2500   2700   2900   3100
Air consumption, kg/h

Figure 1.- Horsepower against air consumption at constant rpm.

2200 rpm

3800

3400

3000

Air consumption, kg/h

2600

2200

1800

1400

Atmospheric conditions-
    pressure = 751.5 mm Hg,
temperature 14°C.

Consumption corresponding to
boost pressure = atmospheric
pressure. P″= 2340 kg/h

500   600   700   800   900   1000   1100
Boost pressure, mm Hg
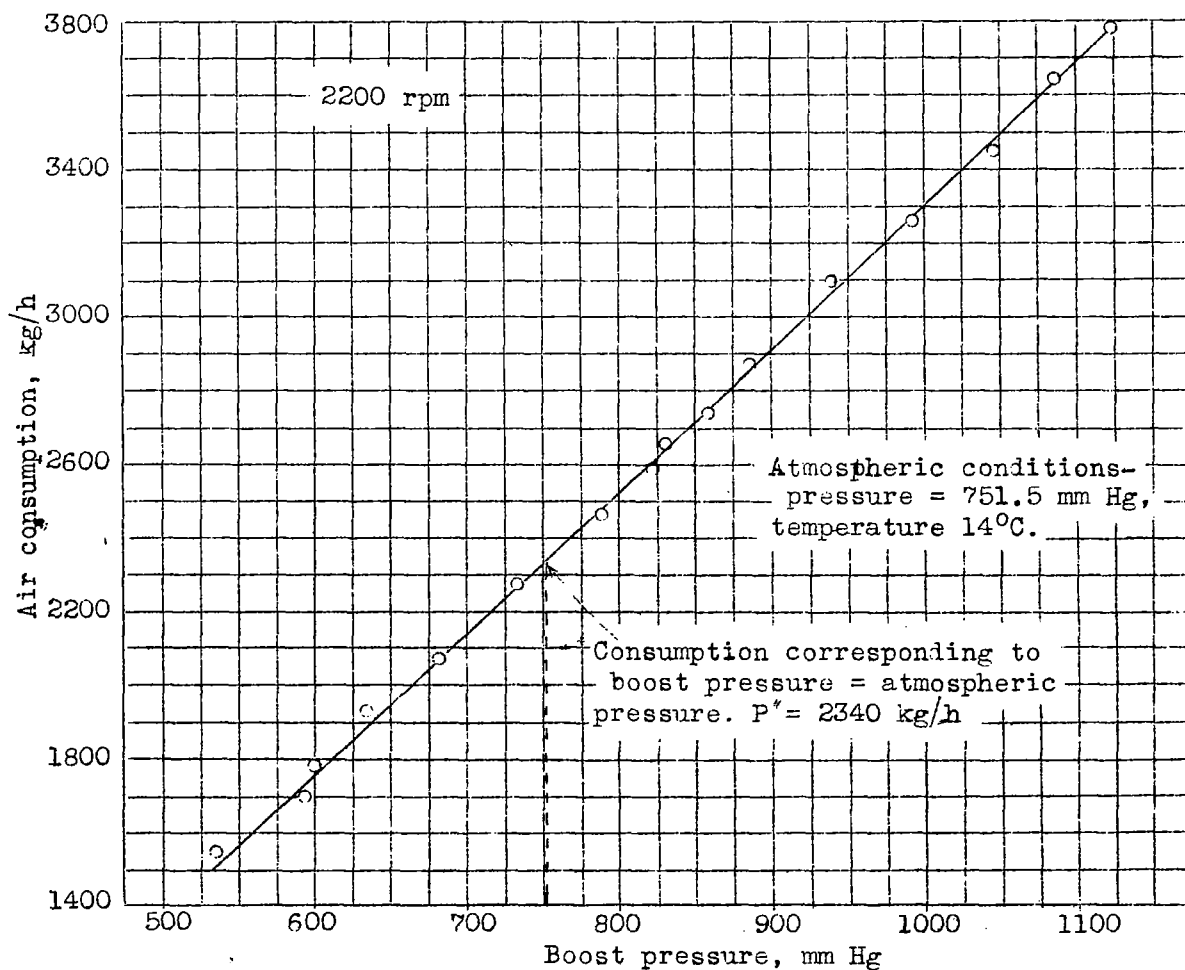
Figure 2.- Air consumption against boost pressure at constant rpm.
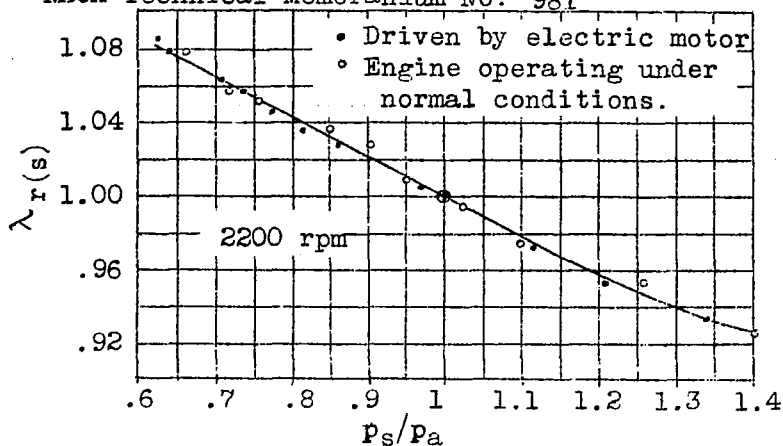
Figure 3.- Coefficient $\lambda_{r(s)}$ against ratio: boost/exhaust pressure.

Figure 4.- Change of compression ratio of supercharger with altitude in standard air and determination of rated altitude for a specified boost pressure.
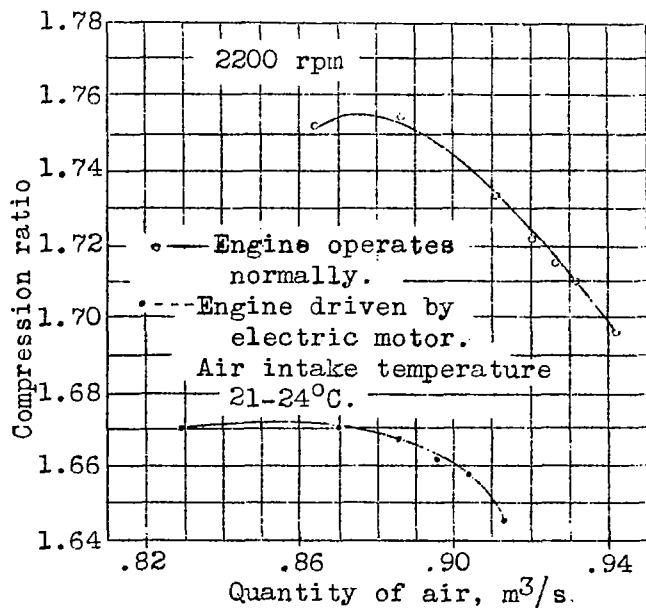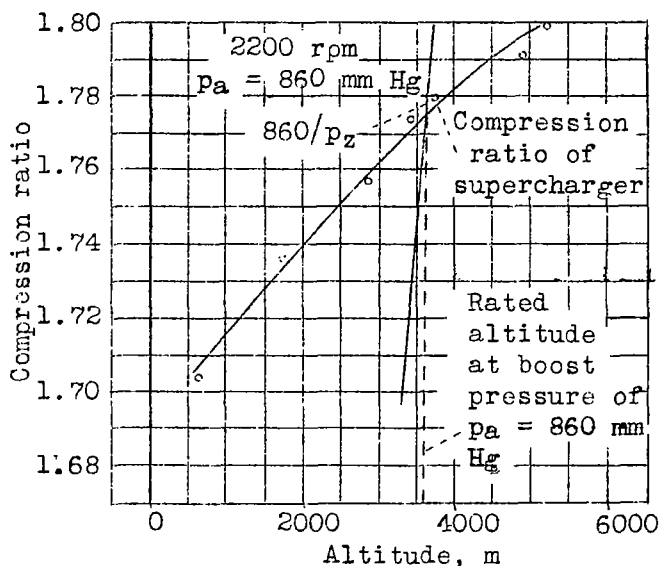

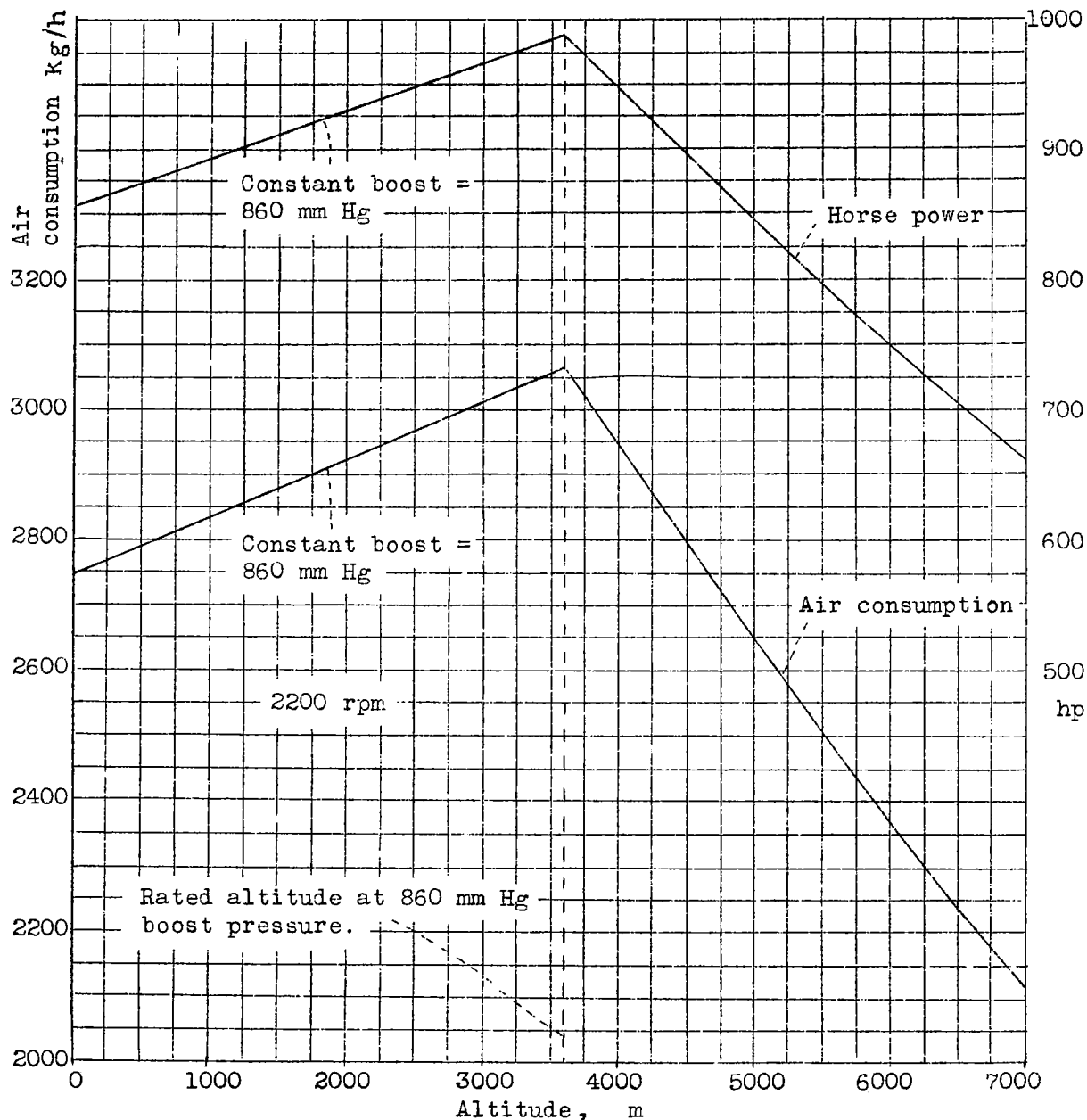Figure 5.- Effect of fuel evaporation on supercharger compression ratio.

Figure 6.- Variation in air consumption and horsepower with altitude
in standard air at constant rpm.